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A selfconsistent semiclassical solution with a throat in the theory of gravity

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We consider the selfconsistent theory of gravity with a vacuum of quantum fields in the framework of the Killing ansatz. The solution, which describes the space–time with a throat, is obtained. The throat's radius has the Planck scale and the space–time is conic far from the throat.

1. Morris, Thorne and Yurtsever [1] have proposed a way for the construction of a time machine. Their proposal was based on the fact that a traversible (without horizon) topological handle or a wormhole with Lorentzian signature of the metric (or shortly a Lorentzian wormhole) exists. Their paper [1] led to many investigations of the physics of Lorentzian wormholes [2–5]. How to avoid the appearance of a singularity and a horizon, was shown in refs. [1,6]; the wormhole's throat has to contain matter fields with a stress–energy tensor $T_{\mu\nu}$ for which the averaged weak energy condition (AWEC) [7] is violated, i.e.,

$$\int_0^\infty T_{\alpha\beta}k^\alpha k^\beta \,\mathrm{d}\lambda<0\,, \tag{1}$$

where $k^\alpha=\mathrm{d}x^\alpha/\mathrm{d}\lambda$ is the tangent vector for a null geodesics passing through the throat. One supposes that the AWEC is realized in quantum field theories. However, this is not known for a solution with a throat in the theory of gravity with quantum fields, because the selfconsistent problem for quantum fields and gravity is very difficult.

The approximate methods for obtaining vacuum expectation values may be used for the selfconsistent investigation. In ref. [8] it was proposed to use the Killing ansatz [9] giving approximate values of the vacuum expectation values of a stress–energy tensor $T_{\mu\nu}$ in static space–times. In this Letter the solution with a throat will be obtained in the framework of the Killing ansatz.

We take the metric signature as $(-,+,+,+)$; the Riemann tensor is determined as $R^\mu{}_{\nu\alpha\beta}=\partial_\alpha\Gamma^\mu_{\nu\beta}+\dots$; $R_{\mu\nu}=R^\epsilon{}_{\mu\epsilon\nu}$. Planck's units $c=\hbar=G=1$ are used.

2. Consider the Killing ansatz [9]. This method consists in constructing an approximate expression $T_{\mu\nu}$ for the vacuum expectation value $\langle T_{\mu\nu}\rangle^{\rm ren}$ using the Riemann tensor, the Killing vector and their covariant derivatives. The tensor $T_{\mu\nu}$ is constructed so that the conservation law $T_\mu{}^\epsilon{}_{;\epsilon}=0$ and the conformal trace anomaly $T^\epsilon{}_\epsilon=\langle T^\epsilon{}_\epsilon\rangle^{\rm ren}$ is fulfilled. In addition a correct scaling transformation behavior of $T_{\mu\nu}$ is necessary. Then the following expression can be obtained for $T_{\mu\nu}$ [#1]:

[#1] Note that the coefficient before $Z_{8\,\mu\nu}$ is $\frac{1}{18}$ but not $\frac{1}{12}$ as in ref. [9]. It may be verified that it is this coefficient that provides a correct conformal trace anomaly.